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PLASTIC-STRAIN EVOLUTION FOR CYCLIC LOADING BASED ON THE EQUATIONS OF THE FIELD THEORY OF DEFECTS

N. V. Chertova and Yu. V. Grinyaev

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A relation governing the plastic-strain evolution under applied stresses is obtained within the field theory of defects to analyze the specific features of deformation under sign-varying cyclic loading. The effect of the applied stress amplitude, loading frequency, and cycle skewness on the deformation process under uniaxial loading conditions is studied. Specific features of the plastic-strain evolution in a stable deformation process are considered, and the time to failure of the system in an unstable process is determined.

Key words: *field theory of defects, plastic strain, cyclic loading.*

Introduction. The behavior of a deformable solid in an elastic region is uniquely determined by reversible elastic strain. Beyond the elastic limit, the experimentally measured quantity is the total strain, which is generally written in continuum mechanics as a sum of elastic, viscous, and plastic components. The last two terms describe the irreversible strain, which can be considered within the continuum theory of defects [1, 2] taking into account imperfection of the crystal lattice. The basic equations of this theory are kinematic identities of an elastic continuum with defects.

In the present paper, we use the equations of the field theory of defects [3, 4], which comprises geometrical relations of an elastic medium with defects and equations governing the dynamics of the ensemble of defects. These equations first written on the basis of the gauge approach [4] and modified in [5, 6] can be used to study the evolution of the defect structure of a material and predict its behavior for various loading conditions. On the basis of the equations of the field theory of defects, the special features of creep were considered for constant and monotonically varying stress [7–9], and the effect of the strain rate on the loading curve of materials was studied [10]. Special features of sign-varying cyclic loading, which often occurs in service of various structures, machines, and mechanisms in transport, rocket and aviation engineering, etc., are considered in the present paper.

Calculation Model. According to [3, 4], the system of equations of the field theory of defects can be written as

$$B\,\partial_k I_{ki} = -P_i^{\rm eff}, \qquad \partial_k \alpha_{ki} = 0, \tag{1}$$
$$e_{ikl}\,\partial_k I_{lj} = \frac{\partial \alpha_{ij}}{\partial t}, \qquad C e_{ikl}\,\partial_k \alpha_{lj} = -B\,\frac{\partial I_{ij}}{\partial t} - \sigma_{ij}^{\rm eff},$$

where α is the dislocation-density tensor, I is the tensor of dislocation-flux density, $P^{\rm eff}$ and $\sigma^{\rm eff}$ are the effective pulse and stress, respectively, B and C are constants, and e_{ikl} is the Levi-Civita tensor. The characteristics of the translational-defect field, density tensor α, and flux-density tensor I are given by

$$\alpha = -\frac{db}{dS} = -\frac{d[u]}{dS}, \qquad I = -\frac{db}{dc} = -\frac{d[u]}{dc}.$$

Here dS is the oriented unit area, c is the closed contour bounding the area dS, and $[u]$ is the displacement discontinuity equal to the total Burgers vector b of dislocations intersecting the area dS. In terms of mechanics of deformable bodies [1], the defect-field parameters can be written as

Institute of Strength Physics and Material Science, Siberian Division, Russian Academy of Sciences, Tomsk 634021; chertova@ispms.tsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 3, pp. 112–118, May–June, 2006. Original article submitted May 16, 2005; revision submitted July 20, 2005.

$$\alpha_{ij} = -e_{ikl}\,\partial_k\beta_{lj}, \qquad I_{ij} = -\frac{\partial\beta_{ij}}{\partial t}, \tag{2}$$

where β is the plastic distortion tensor. Thus, an arbitrary distribution of plastic distortion determines a certain density of dislocations, and its variation with time determines the flux of defects.

In accordance with the model considered, the effective stress and pulse are given by

$$P_i^{\rm eff} = P_i^{\rm ext} + P_i^{\rm int}, \qquad \sigma_i^{\rm eff} = \sigma_i^{\rm ext} + \sigma_i^{\rm int}. \tag{3}$$

Here $P^{\rm ext}$ and $\sigma^{\rm ext}$ are the external pulse and stresses due to the applied load, respectively; $P^{\rm int}$ and $\sigma^{\rm int}$ are the internal pulse and stresses determined by the defects of the material, respectively. As was shown in [5], the quantities $P^{\rm int}$ and $\sigma^{\rm int}$ can be expressed in terms of the characteristics of the defect field:

$$\sigma_{ij}^{\rm int} = C(\alpha_{ki}\alpha_{kj} - (1/2)\delta_{ij}\alpha_{kl}^2) - B(I_{ki}I_{kj} - (1/2)\delta_{ij}I_{kl}^2), \qquad P_i^{\rm int} = Be_{ikl}\alpha_{kj}I_{lj} \tag{4}$$

(δ_{ij} is the Kronecker symbol). With allowance for the results of [6], where the dissipative generalization of the model was considered, system (1)–(4) becomes

$$\begin{gathered} Ce_{ikl}\,\partial_k\alpha_{lj} = -B\,\frac{\partial I_{ij}}{\partial t} - C\Big(\alpha_{ki}\alpha_{kj} - \frac{\delta_{ij}}{2}\,\alpha_{kl}^2\Big) - B\Big(I_{ki}I_{kj} - \frac{\delta_{ij}}{2}\,I_{kl}^2\Big) - \eta I_{ij} - \sigma_{ij}^{\rm ext}, \\ B\,\partial_k I_{ki} = -Be_{ikl}\alpha_{kj}I_{lj} - P_i^{\rm ext}, \qquad \partial_k\alpha_{ki} = 0, \qquad e_{ikl}\,\partial_k I_{lj} = \frac{\partial\alpha_{ij}}{\partial t}. \end{gathered} \tag{5}$$

Here η is the viscosity factor.

In a local approximation, which is commonly used in engineering theories, Eqs. (5) yield the expression

$$B\,\frac{\partial I_{ij}}{\partial t} + B\Big(I_{ki}I_{kj} - \frac{\delta_{ij}}{2}\,I_{kl}^2\Big) + \eta I_{ij} + \sigma_{ij}^{\rm ext} = 0, \tag{6}$$

which relates the applied stresses and the tensor of the defect-flux density. This equation describes the evolution of the plastic strain (2) under the action of external, viscous, and internal stresses. In the case of uniaxial deformation, Eq. (6) is written in the dimensionless variables $V = -(B/\eta)I_{11}$, $T = (\eta/B)t$, and $S = (B/\eta^2)\sigma_{11}$ as

$$\frac{dV}{dT} = \frac{V^2}{2} - V + S. \tag{7}$$

Here $S = S_a \sin(\omega T) + S_0$ (for cyclic loading), V is the longitudinal plastic-strain rate, S_a is the amplitude of the alternating stress, ω is the frequency of external excitation, and S_0 is the mean or static stress determined by multiplying the stress amplitude S_a by the symmetry factor of the cycle a. For plastic strains, Eq. (7) becomes

$$\frac{d^2E}{d^2T} = \frac{1}{2}\Big(\frac{dE}{dT}\Big)^2 - \frac{dE}{dT} + S_a(\sin(\omega T) + a). \tag{8}$$

Results and Discussion. Before we describe and discuss the results, we have to give some commonly available information on cyclic deformation of materials. In many studies addressing the sign-varying cyclic deformation, it was shown that the dependence of elastoplastic properties of a material on the number of loading cycles should be taken into account in both local regions and the specimen as a whole [11, 12]. With respect to variation in the properties under soft and stiff loading conditions, materials are classified into three main types: cyclically stable, cyclically hardened, and softening materials [13]. Materials are referred to as cyclically stable if their resistance to repeated strain does not depend on the number of loading cycles. This means that the number of cycles has no effect on the main parameters of the process: modulus of elasticity, elastic and yield limits, and secant and tangent moduli. With an increase in the number of loading cycles, the strain resistance increases for cyclically hardened materials and decreases for cyclically softened materials. To reveal the special features of variation in stresses and strains for cyclic loading conditions, one has to know the stress-strain relation (the equation of state of the material) after each loading cycle. For this purpose, the kinetics of variation and accumulation of plastic strain in a cycle and from cycle to cycle is to be studied with allowance for the shape and loading parameters. This analysis can be carried out within the model proposed.

The numerical solutions of Eqs. (7) and (8) were obtained and analyzed for different values of external excitation parameters. The stress amplitude S_a was varied from 10^{-3} to 10^3, the frequency ω was varied from 0 to 10^3, and the coefficient a determining the static stress took the values $a = 0$, 0.5, 1.0, and 2.0, which correspond to symmetric ($a = 0$) and asymmetric ($a = 0.5$) tensile–compressive cycling and to pulsed ($a = 1$) and asymmetric

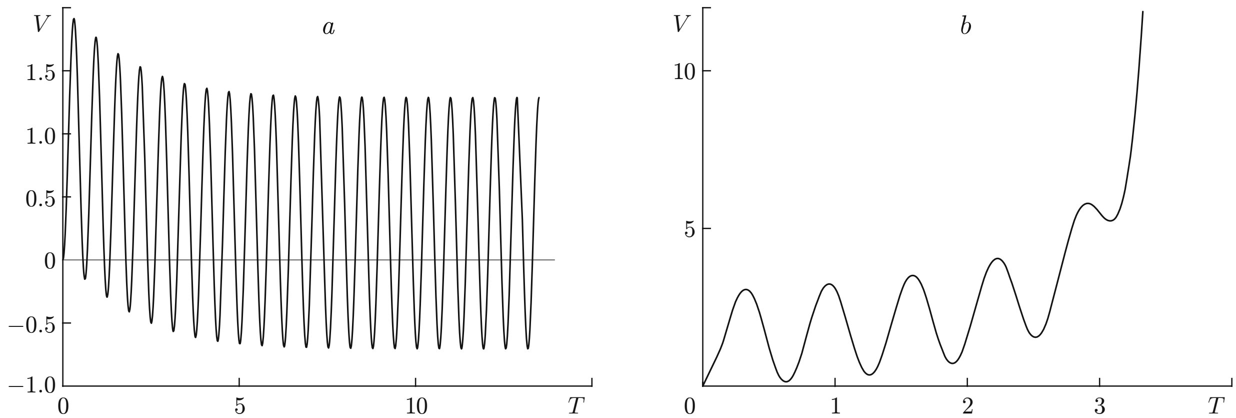

Fig. 1. Evolution of the plastic-strain rate: (a) stable solution ($S_a = 10$ and $\omega = 10$); (b) unstable solution ($S_a = 15$ and $\omega = 10$).

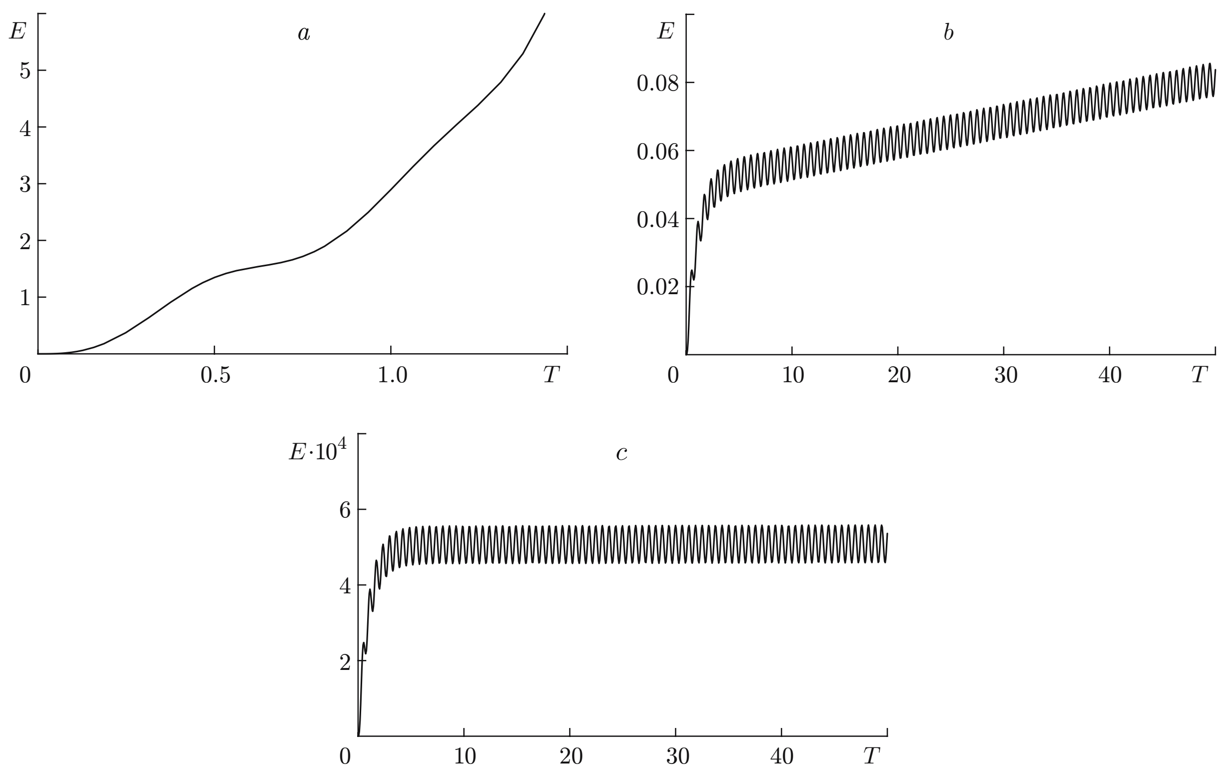

Fig. 2. Plastic-strain dynamics of for different stress amplitudes: $S_a = 20$ (a), 0.5 (b), and 0.005 (c).

($a = 2$) tension cycling. Forrest [14] pointed out that the ratio of static and cyclic loads is one of many factors, such as stress raisers and dimensions of the system deformed, that affect the fatigue resistance. It is known that scale effects due to the influence of system dimensions are demonstrated under bending and torsion of smooth specimens and are little pronounced under extension [12]. In the presence of a stress raiser, the effect of system dimensions is manifested for all loading conditions. Using the present model, one fails to study the effect of system dimensions and stress raisers but can reveal the role of skewness.

We consider a symmetric loading cycle ($a = 0$) such that maximum tensile stress is equal to the maximum compressive stress. The results obtained by solving Eqs. (7) and (8) show that there exists a critical stress amplitude S^* (in our case, $S^* \approx 0.5$), which separates the regions of stable and unstable deformation processes. Figure 1 shows the typical dependences of the plastic-strain rate on time for stable and unstable deformation processes. For

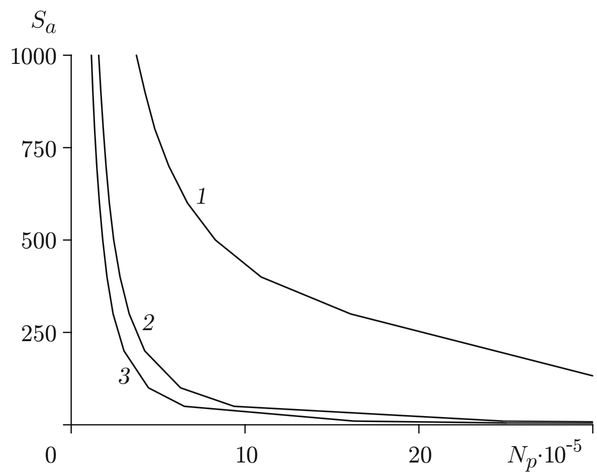

Fig. 3

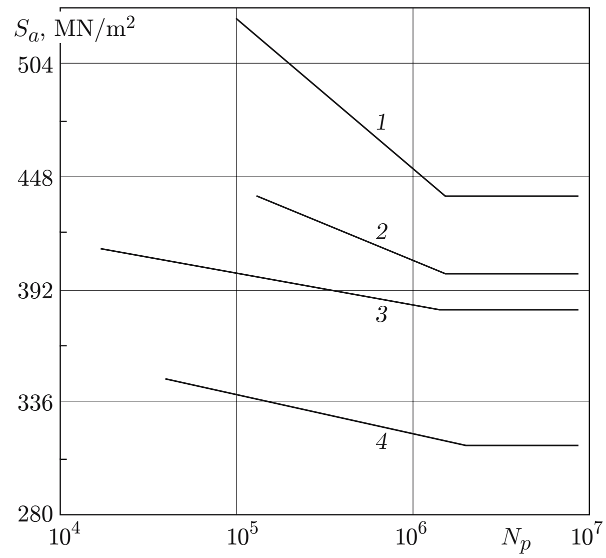

Fig. 4

Fig. 3. Wöhler curves obtained for different values of cycle skewness ($\omega = 100$): $a = 0$ (curve 1), 0.5 (curve 2), and 1 (curve 3).

Fig. 4. Experimental fatigue curves for different values of the applied static stress: $S_0 = 0$ (curve 1), 140 (curve 2), 280 (curve 3), and 420 MN/m^2 (curve 4).

$S_a \leqslant 0.5$, the material is cyclically stable for all frequencies of external excitation. For $S_a > 0.5$, for each stress amplitude there is a frequency ω_* above which the material deformation is cyclically stable. With an increase in the stress amplitude S_a, the frequency ω_* increases.

Figure 2 shows the plastic-strain evolution for the frequency $\omega = 10$ and different stress amplitudes. The curves $E(T)$ obtained for $S_a = 20$, and 0.5 (Fig. 2a and b) are seen to describe the behavior of a cyclically unstable material, whereas the curve $E(T)$ obtained for $S_a = 0.005$ (Fig. 2c) corresponds to the behavior of a cyclically stable material. The deformation processes shown in Fig. 2a and b are characterized by the fact that the plastic strain increases or is accumulated in the material after each loading cycle. Troshchenko [12] believed that both hardening and softening processes can be related to plastic-strain accumulation in local regions. In Fig. 2c, the plastic strain remains unchanged after a certain cycle, which indicates that the characteristics of the loading diagram are constant and the deformation process is cyclically stable. An analysis of results for different loading cycles ($a = 0.5$, 1.0, and 2.0) shows that two deformation regimes may occur: cyclically stable and unstable regimes.

Fatigue tests are usually performed to determine the dependence of the stress amplitude S_a on the number of cycles before the failure N_p for a specified value of the mean stress. Graphically, this dependence is represented by Wöhler's curve [11]. Similar curves can be obtained using the model proposed. For the unstable deformation regime, the moment of failure is determined by unlimited strain growth, which corresponds to the known maximum plastic-strain fracture criterion. The solutions of Eqs. (7) and (8) allows one to obtain the time to failure for a specified stress amplitude, loading frequency, and skewness, which determines the constant stress. Given the fatigue life of a system and loading frequency, one can determine the number of cycles to failure and plot the stress amplitude S_a versus the number of cycles to failure N_p for different values of cycle skewness and $\omega = 100$ (Fig. 3). For comparison, Fig. 4 shows the experimental data of [15], which imply that the constant tensile stress affects the dependence $S_a(N_p)$ for alloyed steel at room temperature. In the case of a stable deformation regime, the maximum allowable (critical) strain should be specified additionally.

Figure 5 shows the endurance (fatigue) limit as a function of the number of cycles per unit time n. The fatigue limit is defined as a stress value below which no failure occurs regardless of the number of loading cycles. The results are in qualitative agreement with experimental data, which show that the fatigue limit increases monotonically with the loading frequency [12, 14].

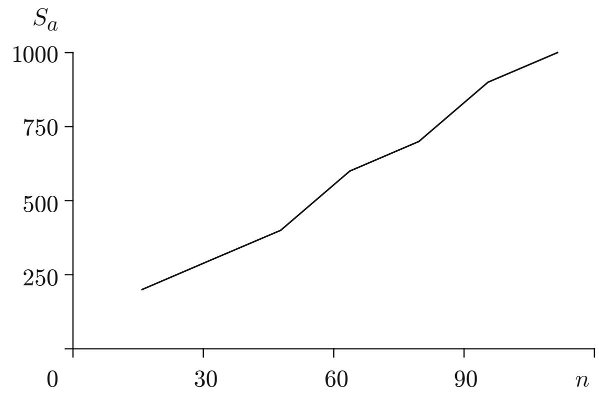

Fig. 5. Fatigue limit versus loading frequency ($a = 0$).

Conclusions. An analysis of the plastic-strain evolution under cyclic loading with the use of equations of the field theory of defects shows that two deformation regimes (stable and unstable) can occur, depending on the loading parameters of the model: amplitude of applied stresses, loading frequency, and skewness. Since relations determining the conditions of transition from one deformation regime to the other contain material constants, a certain treatment of the material that modifies the material properties and changes the material constants can be responsible for the transition from one deformation regime to the other under identical conditions of external excitation.

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